Laboratory 3

Out-of-Plane Beam-Column Bending with Stereo DIC Measurements using VIC-EDU

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I. OVERVIEW

The purpose of this laboratory is to utilize optical imaging systems with stereo digital image correlation software VIC-EDU within the educational measurement system to make full-field, non-contacting surface deformation measurements for a beam-column specimen with simple end supports that is subjected to a compressive load. As part of this laboratory, students will be provided with the basic theoretical background. To obtain the measurements, students will (a) prepare the specimen surface and apply a usable, high-contrast speckle pattern on the surface of the test specimen, (b) calibrate the VIC-EDU system to acquire images that can be used for stereo analysis, (c) configure the VIC-EDU measurement head to obtain image pairs during mechanical loading, (d) acquire and store images during mechanical loading of the specimen, (e) perform post-processing to obtain results using the stereo images and (f) compare the experimental results to theoretical or hybrid experimental-theoretical solutions and discuss your findings.

II. BEAM-COLUMN THEORY

A specimen for which the well-known Euler-Bernoulli beam theory is applicable is shown schematically in Fig. 1 (see pg. 2), where the theoretical formulation generally requires that $L >> h >> t$. Typical ratios for $\frac{h}{L}$ and $\frac{t}{h}$ are on the order of 0.5, 0.33, 0.25 or even lower, depending upon experimental limitations for the application. If these conditions are met, and the deformations are relatively small, then the bending moment is related to the curvature of the beam in a manner that is similar to Eq. 1 in Lab 2. In this laboratory experiment, there will be an axial compressive load and the bending moment will be about the Y_c as shown in Fig. 1.

During loading, the beam will deflect in the Z_c direction, as shown schematically in Fig. 1. Also shown in Fig. 1, the beam slope is positive so that the curvature is negative along the x_c direction. Since $M_{y_c}(x_c)$ is positive as shown, the corresponding moment-curvature equation is written

$$EI_{y_c y_c}\frac{d^2w}{dx_c^2} = -M_{y_c}(x_c) \quad (1)$$

where E is the elastic modulus (units of Pa or psi) of the material; $I_{y_c y_c}$ is the area moment of inertia of the beam cross-section about the centroidal Y_c axis; $w(x_c)$ is the out-of-plane displacement of the centroidal $X_C - Y_C$ surface at distance x_c along the beam; $M_{y_c}(x_c)$ is the internal reaction moment about the Y_C axis as a function of the distance, x_c, along the beam. The direction shown in Fig. 1 for M_{Y_C} is assumed to be positive. In addition to the internal reaction moment, there is an axial compressive force, P in Fig. 1, on the $Y_C - Z_C$ cross-section passing through the centroid, C, of the beam-column cross-section.

The specimen provided with the VIC-EDU system has an estimated modulus of elasticity, E ~ 6 GPa. As described in a NOTE in Section VI of this Lab, there may be variability in E due to manufacturing differences for the as-supplied specimen material.

For the rectangular beam shown in Fig. 1, the location of the centroid C in the undeformed position is $(x_{c,}\frac{h}{2},\frac{t}{2})$ relative to the bottom left corner of the beam. The origin for the centroidal coordinate system can be chosen anywhere along the beam specimen. However, in practice, the origin is oftentimes chosen at one end of the beam specimen and oriented so that x_c is positive along the entire length of the beam.

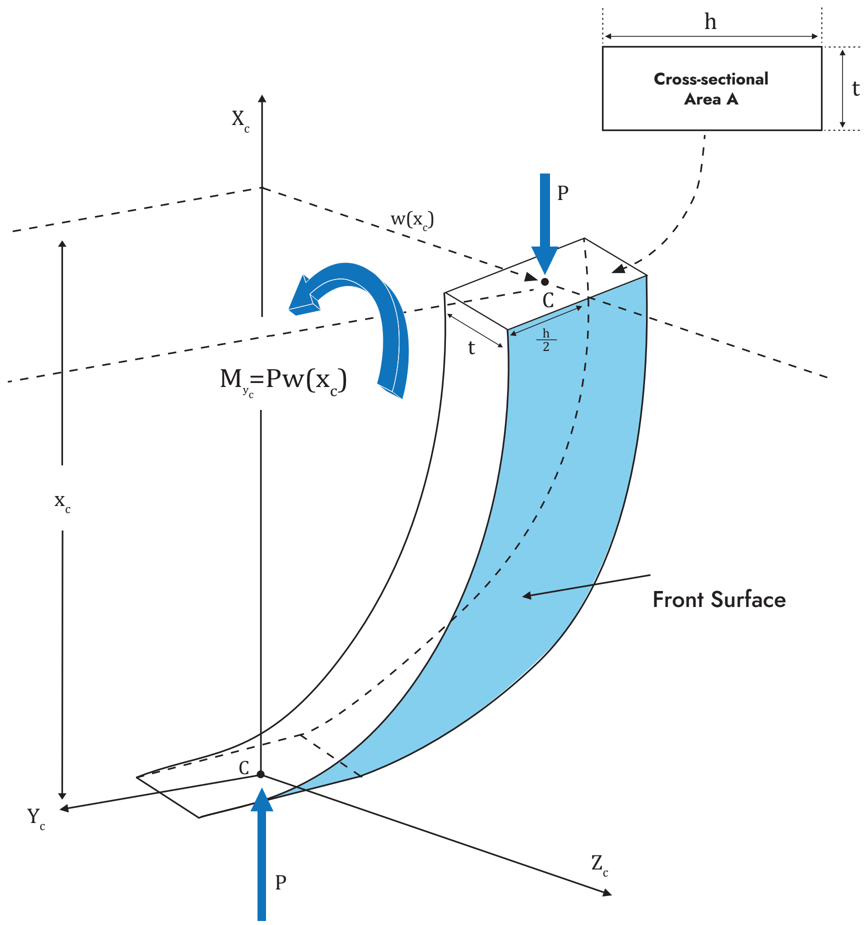

Figure 1: Beam-column undergoing compressive load with out-of-plane deflection in Z_c direction

Specimen dimensions: t = 0.03125 in, h = 1.00 in, L = 4 in, A = 0.03125 in^2

What is unique for a beam-column is that the bending moment M_{y_c} is solely due to the axial load P and the beam deflection, $w(x_c)$, as shown schematically in Fig. 1. If $w(x_c)$ is in the $+Z_c$ direction, then the moment about the Y_c axis is positive. Thus, we have that

(2) $$M_{y_c}(x_c) = P * w(x_c)$$

Combining Eqs. (1) and (2), the beam-column equation is given as;

(3) $$EI_{y_c y_c} \frac{d^2 w}{dx_c^2}(x_c) = -P * w(x_c)$$

Eq. 3 is the well-known Euler buckling equation. The solution to Eq. 3 is

(4) $$w(x_c) = A\cos(\beta x_c) + B\sin(\beta x_c),\ \beta = \sqrt[2]{(P/EI_{y_c y_c})}$$

The amplitude parameters A and B are determined via the specified boundary conditions. In this laboratory, the boundary conditions are best approximated by assuming the following displacement conditions at the ends

$$w(x_c = 0) = 0 \qquad (5)$$
$$w(x_c = L) = 0$$

Eqs. 4 and 5 show that $A = 0$ and that the lowest frequency, $\beta_{min} = \frac{\pi}{L}$. Furthermore, if the beam-column were perfectly straight, the compressive load that initiates buckling is given by $P_{crit} = \frac{\pi^2 EI_{YcYc}}{L^2}$

Of course, if the beam is even slightly bent, then there is no need for the buckling load as the beam already has deformed out-of-plane and will continue to bend out-of-plane with increasing load. If this is true, this is a simple example of a condition known as "post-buckling". In such a case, the final solution for the displacement is

$$\mathrm{w}(\mathrm{x_c}) = B\,sin\left(\frac{\pi \mathrm{x_c}}{L}\right) \qquad (6)$$

where B is an arbitrary amplitude for the beam that cannot be determined using the applied loading. If the beam deflection is measured at one location, say $w(x_c = \frac{L}{2})$, then the solution can be written;

$$\mathrm{w}(\mathrm{x_c}) = \mathrm{w}\left(\frac{L}{2}\right) * sin\left(\frac{\pi \mathrm{x_c}}{L}\right) \qquad (7)$$

Thus, Eq. (7) shows that the theory predicts the beam shape is a sine function along the length, though the amplitude of the displacement cannot be determined unless it is measured at a point, such as $x_c = \frac{L}{2}$

In addition to Eq. (1), when cross-sections $Y_C - Z_C$ remain planar during mechanical loading, then static analysis of the beam and simple strength-of-materials concepts can be employed to show that there is a normal stress, $\sigma_{\mathrm{x_c x_c}}$ on each cross-section given as;

$$\sigma_{\mathrm{x_c x_c}} = \frac{M_{\mathrm{y_c}}(\mathrm{x_c})\mathrm{z_c}}{I_{\mathrm{y_c y_c}}} - \frac{P}{A} \qquad (8)$$

where P is the compressive load and A is the cross-sectional area of the beam. If the beam is assumed to be loaded only on the top and bottom surfaces in the axial direction (see Figs. 1 and 3b) and the beam is an isotropic, homogeneous, linear elastic material, then the corresponding strains along the x_c direction can be written;

$$\varepsilon_{\mathrm{x_c x_c}} = \frac{M_{\mathrm{y_c}}(\mathrm{x_c})\mathrm{z_c}}{EI_{\mathrm{y_c y_c}}} - \frac{P}{AE} \qquad (9)$$

If the strain, $\varepsilon_{\mathrm{x_c x_c}}$, is obtained experimentally on the front surface (see Fig. 1) at the same location as $w(x_C)$, say at $x_c = \frac{L}{2}$ then Eq. (9) can be re-written to solve for the axial load P as follows:

$$\varepsilon_{\mathrm{x_c x_c}}\left(\frac{L}{2}\right) = \frac{P\mathrm{w}\left(\frac{L}{2}\right)\frac{t}{2}}{EI_{\mathrm{y_c y_c}}} - \frac{P}{AE} = P\,\frac{\mathrm{w}\left(\frac{L}{2}\right)\frac{t}{2}}{EI_{\mathrm{y_c y_c}}} - \frac{1}{AE} \qquad (10)$$
$$P = \frac{E\,\varepsilon_{\mathrm{x_c x_c}}\left(\frac{L}{2}\right)}{\left[\frac{6\mathrm{w}\left(\frac{L}{2}\right)}{ht^2} - \frac{1}{th}\right]}$$

Eq. 10 provides a theoretical formulation that can be used to predict the applied compressive load, P; by combining Eq. 10 with surface measurements, the axial compressive load can be predicted. The load obtained using Eq. 10 can be compared directly to the actual applied load as a way to assess the accuracy of the measured strain and out-of-plane displacement.

In contrast to the discussion given in Lab 2, for the post-buckling specimen the theory shows that we can predict the SHAPE of the $w(x_c)$ data but we cannot use the load to predict the *magnitude* of the displacements. Thus, we cannot *a priori* determine whether the experimental displacements are large enough to be measured by the VIC-EDU system ($|w(x_c)| > 0.04mm$).

NOTE **Throughout these labs, we refer to the VIC-EDU system in two ways. The hardware (case, cameras, cords, calibration targets) are referred to as "VIC-EDU" and the software (the special version of VIC-3D and VIC-Snap that are designed for this system) as "VIC-3D (EDU)" and "VIC-Snap (EDU)".**

III. EXPERIMENTAL PREPARATIONS

III. a. Specimen Preparation

Specimen preparations for the beam-column specimen are similar to those described in Lab 2, the VIC-EDU User's Manual [1] and the digital image correlation book [2]. To make accurate DIC measurements using images of a specimen, the specimen must be prepared and the FRONT SURFACE patterned with high contrast speckles having the appropriate size and an approximately random distribution across the field of view. The procedures to do this are detailed in LAB 1 and in Application Note AN-1701 [3]. The steps are briefly summarized.

- Use coarse and fine grit sandpaper to smooth surface and remove unwanted adherent materials.
- Use CSM-2 degreaser or similar cleaning agent to remove any residual oils and metal particles left on the surface by sanding. Repeat this as needed.
- Allow surface to dry and then paint the specimen surface. DO NOT paint specimen in same room where imaging is performed, as it will degrade the optical imaging elements. Typically, surface is initially painted white.
- Use only enough paint to coat surface and minimize reflections; DO NOT OVERPAINT SURFACE.
- After the white paint has dried, apply a random black dot pattern with size of 0.026" (0.635 mm) using the supplied roller and ink pad. Patterning should require approximately 2-3 passes in order to achieve the high contrast 50/50 ratio of white to black that is best. Do not try to align the roller passes, as attempts to move the roller around during patterning will smear or blur the dots, potentially degrading the quality of stereo DIC measurements. When done, image the front surface and check the pattern with VIC-3D (EDU) tools.

III. b. Installing Specimen in Loading System

Once the specimen has been prepared, the specimen can be placed in the load frame. One end of the specimen displaces downward due to the load. The other end of the specimen does not displace, both ends are free to rotate. These end conditions are SIMPLE SUPPORTS.

III. c. VIC-EDU System Preparation

Once the specimen is in the loading frame, the as-suppled tripod should be located in front of the specimen and the VIC-EDU system firmly attached to the tripod mounting head. At this point, the VIC-EDU system should be powered up and oriented to obtain images of the front surface of the specimen. Once the system is operational, the procedures outlined in Lab 1 for focusing the specimen and preparing to acquire images should be re-read and then used to locate the system approximately 0.5m in front of the specimen where focused images can be obtained. At this point, calibration of the system is performed.

III. d. VIC-EDU System Calibration

Once the VIC-EDU system is mounted to the tripod and placed in position where the specimen is in sharp focus, system calibration is performed. Though in principle the entire system could be picked up and moved carefully to a new location for calibration, and then moved back into position for viewing the specimen without affecting the measurements, such movements may introduce changes in the optical imaging system that are not readily apparent and can affect the accuracy of the results. To minimize the potential for such errors, it is recommended that you calibrate in front of the test specimen. There should be enough depth of field to allow for this. Lighting provided within the VIC-EDU measurement head should provide bright, clear images of the as-supplied calibration target.

When performing calibration, it is recommended that the target occupy 80-90% of the field of view. For this experiment, the as-supplied target dot pattern has black dots with a 14mm spacing on a white background. Fig. 4 shows a typical dot pattern imaged by the VIC-EDU system. Furthermore, it is required that all three of the black dots containing a central white dot be within the field of view of both cameras. These dots provide the necessary information for estimating rotation of the planar target.

To perform calibration, the target should be in focus for both cameras. Since the focus is fixed on the VIC-EDU system, the target should be located approximately 0.50m in front of the system. Then, the target is rotated several times and focused images acquired. The rotations should include motions about three separate orthogonal axes to ensure accurate identification of all calibration parameters. Typically, between 25 and 50 calibration images are acquired. Please refer to Lab 1 and the User's Manual for additional information regarding calibration and processing of the calibration images.

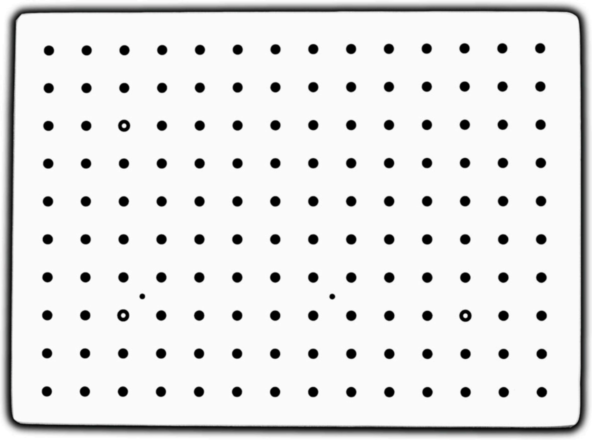

Figure 2: Schematic of a calibration target with 14mm dot spacing

IV. PERFORMING EXPERIMENT WITH IMAGE ACQUISITION

Once calibration is successfully completed in front of the specimen, the target is removed so the front surface of the beam specimen already placed in the loading frame is visible and in focus.

- Read Lab 1 and VIC-EDU User's Manual regarding the procedure for initiating VIC-EDU to acquire and store image pairs for this experiment.
- Acquire several image pairs of the speckled front surface of the specimen in the unloaded state.
- Apply the first increment of load/displacement and acquire two to three pairs of images, recording the image numbers and the corresponding applied load/displacement.
- Continue load/displacement application and image acquisition process until you reach the pre-identified maximum load or displacement for this specimen.
- Terminate the image acquisition process and the experiment via the process described in Lab 1 (e.g., close VIC-Snap (EDU).
- Unload the specimen.

At this point, the experimental process is complete and the deformation data is embedded in the image pairs that were acquired during the experiment. To extract full-field deformation data, the VIC-3D (EDU) software is initiated to analyze the images. Please refer to the detailed description in Lab 1 for how to initiate VIC-EDU and extract displacement and strain data.

V. POTENTIAL SOURCES OF DIFFERENCES BETWEEN PREDICTIONS AND MEASUREMENTS

For the beam-column specimen, the analytical results using Eq. 1 require you to assume specific boundary conditions at $x = 0$ and $x = L$. If the bottom support actually moves slightly outward/inward during loading at one end and has restricted rotation, then these will affect the comparison. If either of these occur, the entire $w(x_c)$ curve can be affected. Such motions are oftentimes difficult to remove from the experimental process and so may affect the comparison of experimental and theoretical displacements.

An additional source of potential errors is the way in which the experimental data is processed. For example, the experimental strains obtained from VIC-EDU use the measured displacements in a region, with a "finite-size strain filter" to estimate the strains at a point. The strain filter has various sizes over which it determines the strain. This process may introduce slight shifts in the strain relative to the actual strain at the corresponding point on the specimen. If $\varepsilon_{x_c x_c}$ is measured at $x = \frac{L}{2}$, then the size of the strain filter can affect the measured strain value and comparisons to theoretical estimates.

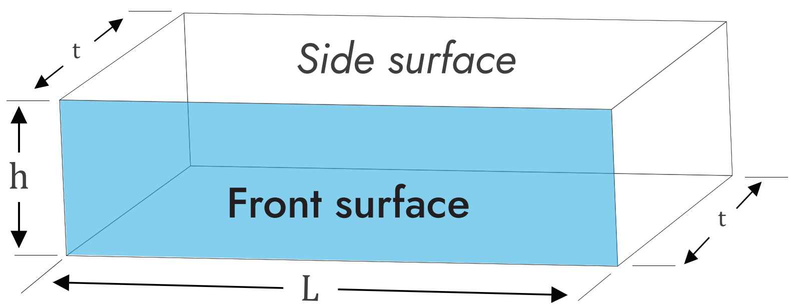

Figure 3a: General schematic of the beam-column specimen to be used with compressive loading to obtain out-of-plane displacements

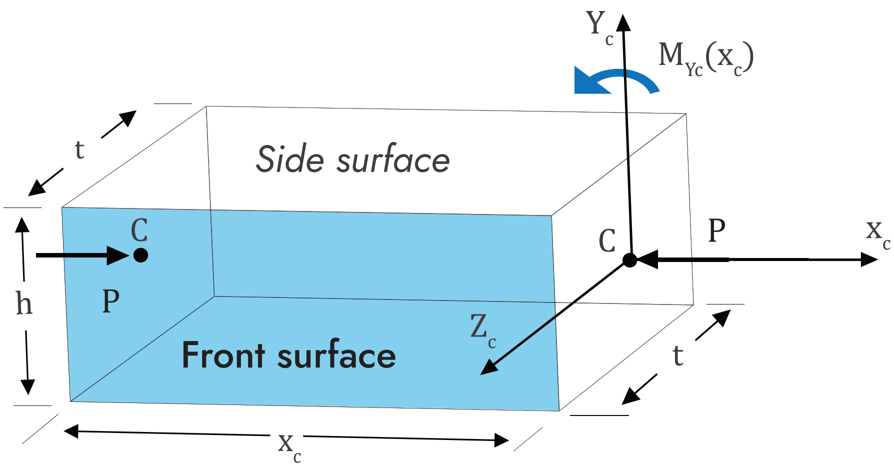

Figure 3b: Beam-column specimen with compressive load and corresponding bending moment. Front surface will deform in Z_c direction causing out-of-plane displacements

VI. LABORATORY RESULTS TO BE PRESENTED

Note: The range on the modulus of elasticity, E, for the test material is 3 GPa ≤ E ≤ 9 Gpa, with the expected value being 6 Gpa.

1. Measure the specimen geometry (L, h, t) several times with a micrometer or similar device and provide a table of the individual results, mean value for each dimension and standard deviation for each dimension.

2. Using the VIC-EDU system, acquire two pairs of test images of the as-patterned specimen after it has been centered, focused and properly exposed, but without any load. Both pairs of images should be taken without moving the specimen. Include one pair of test images for the beam-column specimen in the report.

   - As you are setting up the analysis, fill in the table below to provide a list of all options/parameters used in the analysis. Note that this table should be filled out for all StereoDIC experiments, without exception, to have a record of the analysis parameters being used.
   - Using one pair of stereo images obtained for $P = 0$, plot the initial shape of the specimen, $z(x, y)$, to see whether it is nominally flat or not. Here, it is particularly important to show the initial shape and determine whether the specimen is in a "post-buckled" shape that is bent towards or away from the cameras.
   - Discuss how the measurements are related to the concept of "post-buckled shape".

3. Use two pair of stereo images obtained for P=0 and measure out-of-plane displacement field, $w(x_c, y_c \frac{t}{2})$, of the specimen on the front surface in Fig. 1. For the remainder of this section z= $\frac{t}{2}$ is assumed to be true, so $w(x_c, y_c) = w(x_c, y_c \frac{t}{2})$.

   - Plot $w(x_c, y_c = 0)$ along the length of beam, which is along the x_c-axis in Fig. 1
   - Obtain the mean value for $w(x_c, 0)$ and the standard deviation in $w(x_c, 0)$ along centerline
   - Discuss the results and relationship to CSI estimate of 0.04mm for variability.

4. Assuming the experimental beam-column boundary conditions are consistent with Eq. (5), the beam-column displacement data is theoretically given by Eq. (7).

   - Use VIC-3D (EDU) analysis software to extract the out-of-plane displacement data along the x_c-axis direction in Fig. 1, which is along the beam length that passes through the centroid location for $0 \le x_c \le L$ for N compressive loads.
   - Extract $w(x_c = \frac{L}{2}, y_c = 0)$ from the measurements for each load and insert into Eq. (7) for each load.
   - Plot Eq. (7) vs the experimental data on $0 \le x_c \le L$ for compressive loads P_i, i = 1, 2, ..., N.

5. Using the VIC-3D (EDU) analysis software, extract the strain $\varepsilon_{x_c x_c}$ along the x_c-axis direction on the front surface in Fig. 1, which is along the beam length, for $0 \le x_c \le L$ for N compressive loads (same line as used in items 2-4).
   - Extract $\varepsilon_{x_c x_c}(x_c = \frac{L}{2}, 0)$ from the front surface strain measurements for each load.
   - Using the measured values of $w(x_c = \frac{L}{2}, 0)$, $\varepsilon_{x_c x_c}(\frac{L}{2}, 0)$, t, L, h for each load, calculate the compressive load using Eq. (10).
   - Plot the ratio $[\frac{P_{Eq.(10)}}{P_{true}}]\ vs\ P_{true}$

Undergraduate Students Only

6. Discuss results from 1., 2., and 3.

Lab 3 - Stereo DIC Image Analysis Parameters

Region of Interest (ROI) (pixels x pixels)	
Field of View (pixels x pixels)	
Digital magnification (pixels/mm)	
Subset size (pixels)	
Subset spacing (pixels)	
Correlation criteria used	
Subset weighting	
Interpolation method	
Consistency threshold (pixels)	
Confidence margin (pixels)	
Matching threshold (pixels)	
Strain metric type	
Strain filter size (N x N $pixels^2$)	
Strain filter type	
Auto-plane fit	Yes or No

Graduate Students

7. This part compares the strains computed using VIC-3D (EDU) to the values obtained using a simple formula from undergraduate solid mechanics. You already have determined strain with VIC-3D (EDU) along the x-axis in Fig. 1 through the centroid.
   - Along the x_c-axis direction in Fig. 1, output the axial displacement, $U(x_c, 0) = U(x_c)$, obtained for each subset at the center of the front surface.
   - For each neighboring value, compute $E_{x_c x_c} = \frac{U(x_c + \Delta x_c) - U(x_c)}{\Delta x_c}$ for all the points along the line for each load.
   - Plot $\varepsilon_{x_c x_c}$ and $E_{x_c x_c}$ along $0 \le x_c \le L$ for N compressive loads.
8. The results obtained by these methods will not agree well. However, if you calculate strain $e_{x_c x_c}$ using the following approach, then it should agree with $\varepsilon_{x_c x_c}$.
   - Along the line, define the undeformed coordinates of a point as (X_c, Y_c, Z_c) in the unloaded state for each point of interest.
   - For each pair of neighboring subsets, compute the undeformed distance between the 2 subsets as $\delta = \sqrt[2]{(X_{c_{i+1}} - X_{c_i})^2 + (Y_{c_{i+1}} - Y_{c_i})^2 + (Z_{c_{i+1}} - Z_{c_i})^2}$
   - For each pair of neighboring subsets and load P_i, let $(x_{c_i}, y_{c_i}, z_{c_i})$ denote the deformed position of the originally undeformed point $(X_{c_i}, Y_{c_i}, Z_{c_i})$. Compute the deformed length of the distance between the same subsets as $\Delta = \sqrt[2]{(x_{c_{i+1}} - x_{c_i})^2 + (y_{c_{i+1}} - y_{c_i})^2 + (z_{c_{i+1}} - z_{c_i})^2}$ for all the points along the line for each load.
   - Compute $e_{x_c x_c} = \frac{\Delta - \delta}{\delta}$ for all subset pairs.
   - Plot $\varepsilon_{x_c x_c}$ from the software and $e_{x_c x_c}$ along $0 \le x_c \le L$ for N compressive loads.

If done correctly, these results will agree well and this should be evident in the comparison plot. Please discuss these results and explain the difference between $\varepsilon_{x_c x_c}$ and $E_{x_c x_c}$.

9. If the maximum applied load is P_{max}, show the full-field plot of (x_c, y_c) on the entire front surface for $0 \le x_c \le L$ for $0, \frac{P_{max}}{5}, \frac{2P_{max}}{5}, \frac{3P_{max}}{5}, \frac{4P_{max}}{5}$ and P_{max}.
   - Is the strain field constant along the length or does it vary as you move from 0 to L?
     - If it varies, how does it vary?
     - Please explain why the observations make sense using the theoretical results from Eqs. (1-10).
       - Does the strain field vary across the width of the specimen?
     - Please explain your findings based on Eqs. (1-10).

10. Discuss results in 1., 2., 3., 4. and 5.

NOTES

VII. REFERENCES

1. VIC-EDU User's Manual, Correlated Solutions Incorporated, www.correlatedsolutions.com
2. MA Sutton, JJ Orteu, and HW Schreier, Image Correlation for Shape Motion and Deformation Measurements; Theory and Applications, Springer (2009) ISBN: 978-0-387-78746-6.
3. Application Note AN-1701:Speckle Pattern Fundamentals, http://www.correlatedsolutions.com/support/index.php?/Knowledgebase/Article/View/80/1/speckle-pattern-fundamentals.